Limit to the detection of Glass patterns in the presence of noise

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A method is developed for quantifying the strength of the moiré effects known as Glass patterns. Unpaired randomly placed dots are added to the pattern while the discriminability d' of the degraded pattern is determined in a yes–no test. For a given discriminability the number of pairs required increases in direct proportion to the number of random dots. A model is developed based on the ideal discrimination of an excess of oriented pairs. Results conforming to Weber's law are predicted; the dependence of d' on the amount of noise and the number of point pairs is also predicted. A numerical constant derived from the model provides a measure of the strength of the moiré effect of a chosen pattern. Note that, for this task, statistical considerations predict Weber's law, not the square-root law, as a limit, and this result holds whenever second-order structure is detected in the image.

INTRODUCTION

One of the more eye-catching and unexplained phenomena of modern psychophysics is the streaky appearance of a pattern of oriented point pairs. These patterns, first described by Glass,[1] are created by taking a field of random dots, transforming it in some way (e.g., by rotation, translation, or dilation), and superimposing the transformed copy upon the original (Fig. 1). The result is a pattern of randomly placed dot pairs oriented in the direction of the local transformation, but the appearance is that of a streaky wood grain oriented in the direction of the paired dots. One reason why these patterns are particularly interesting is as follows.

It goes without saying that most of what we perceive results from some sort of patterning of the physical stimulus, but in this case the pattern is unusually simple. This simplicity is not, however, subjectively obvious: most observers, when they first see the streaky appearance, find it difficult to believe that it results from a simple pairwise replication of the dots. This prompts the question whether other perceptual effects such as motion, depth, and symmetry might not be built up from the detection of pairwise regularities, and it renews interest in the role of autocorrelations and cross correlations in perception. There are, of course, hierarchies of mechanisms in the brain, but is it necessary to postulate anything more than the detection of paired conjunctions at any one level to explain perception?

A variety of subjective observations has been made about these patterns. For example, perceptibility appears to decrease with increasing separation between the paired dots[1]; similarly, randomly occurring chains of dot pairs seem to aid detection.[2] These observations are either purely impressionistic or based on attempts by the subject to grade parallelism by detailed inspection. In this paper we apply signal-detection theory to quantify the strength of the moiré effect of Glass patterns, and we relate the quantitative performance observed to a model related to that of Stevens[2] that extracts the local orientation of point pairs.

MATERIALS AND METHODS

Patterns were generated on a Motorola VME10 computer based on a 68000 series microprocessor and were displayed on a Hewlett-Packard x–y monitor (Model 1332A) driven by a digital-to-analog converter. The system is capable of displaying 2000 points in 15 msec. The computer randomly generated with a probability of 1/2 either (1) an oriented signal consisting of a Glass pattern of a specified type (e.g., translation in one direction through 9 min of visual angle with a given number of point pairs or (2) an unoriented pattern consisting of an equal number of randomly oriented point pairs with the same spacing between dots. Noise, consisting of a specified number of randomly placed (unpaired) points, was then added. The local densities of the patterns from the two classes were the same; they could be distinguished only by the orientation of point pairings. Within a pattern, the spacing between point pairs was held constant; in particular, in rotation patterns the points were rotated through a distance subtending a constant angle at the eye rather than a constant angle at the center of the pattern.

Observers were seated 1 m from the display, where the patterns were disks subtending 6 deg of visual arc. Visual acuity was corrected to 20/20 for all subjects. Observers were presented with the patterns in a yes–no test. A fixation dot appeared at the center of the disk for 1 sec, and then the generated pattern was presented for 150 msec. The observer signaled his choice (oriented or random) and was immediately given the correct answer. The observer could sample patterns from both classes and practice for as long as desired before starting a run of experimental trials. The results presented here are those obtained with the three authors as observers.

We assume that oriented and random patterns both produce internal representations that can be described as real variables normally distributed with different means u_o and u_r (representing their respective degrees of orientedness) but equal variance σ, determined by the added noise and th

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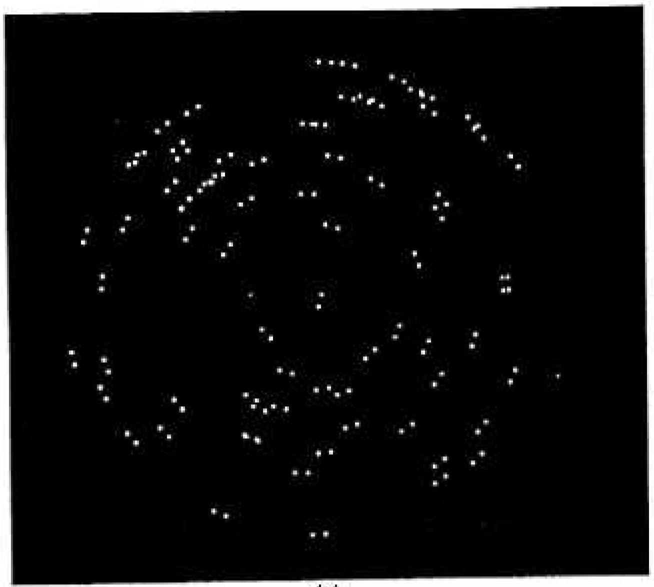

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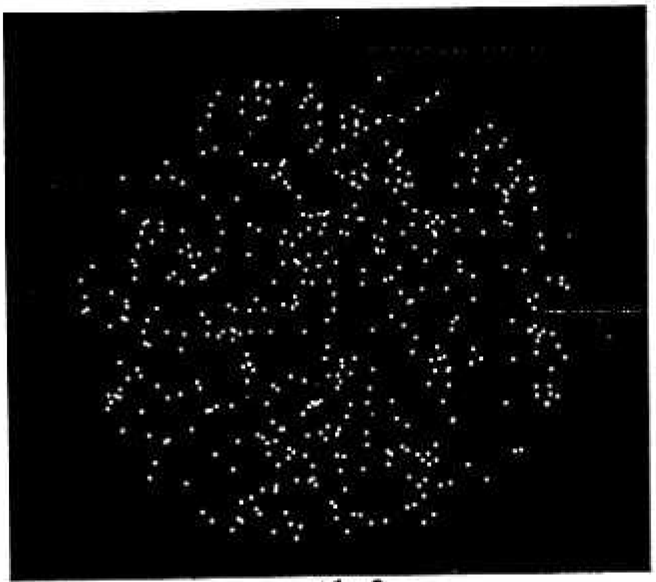

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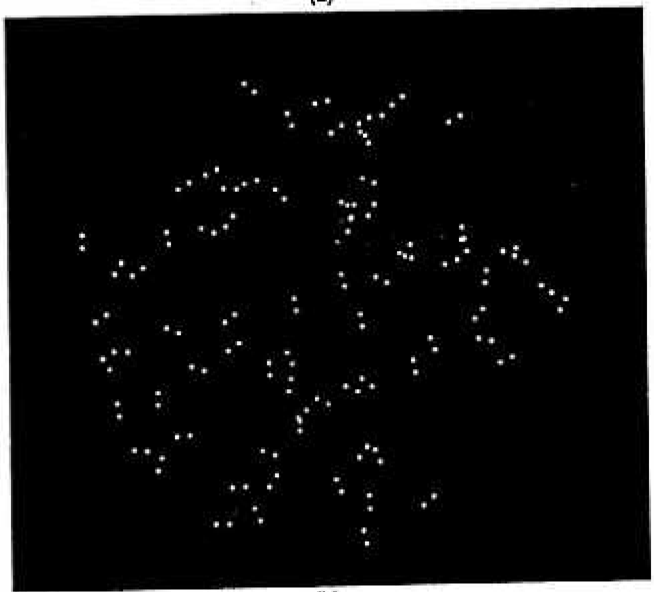

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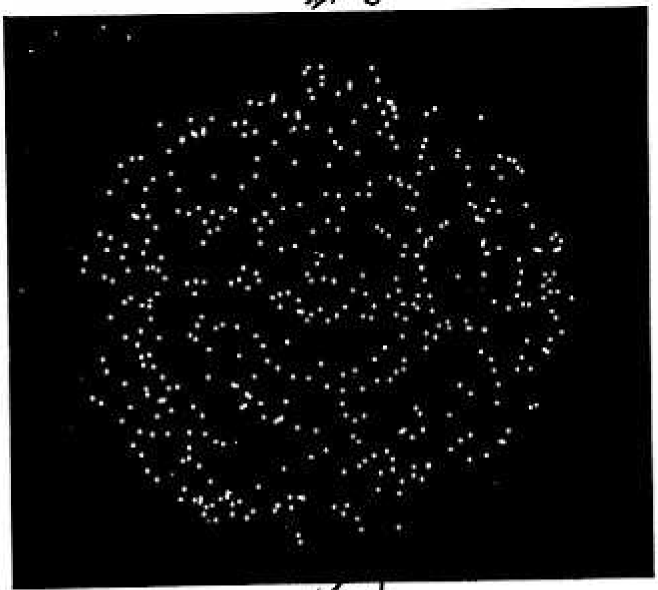

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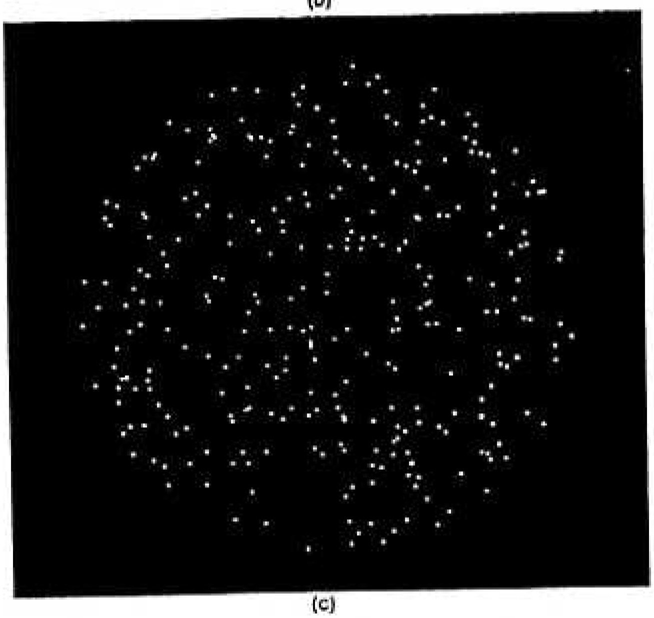

(c)

Fig. 1. (a) Glass pattern with 75 point pairs formed by rotation through a distance that would subtend 9 arcmin at the normal viewing distance, where the patterns subtend 6 deg at the eye. (b) Set of 75 randomly oriented pairs with 9 min of separation. (c) 300 randomly placed points. (d) The oriented pairs of (a) with the random points of (c). (e) The randomly oriented pairs of (b) with the random points of (c). The streaks visible in (a) can still be detected in (d) despite the added noise.

noise internal to the visual system. d' is defined as

$$d' = (u_s - u_r)/\sigma, \qquad (1)$$

and it can be obtained from the frequency of the subject's errors. Under the above assumptions, d' can be calculated from the error rate in a yes–no test:

$$d' = P^{-1}(a) + P^{-1}(b), \qquad (2)$$

where a is the fraction of oriented patterns mislabeled as random by the observer, b is the fraction of random patterns mislabeled as oriented, and $P^{-1}(x)$ is the inverse function of the Gaussian probability function

$$P(y) = \frac{1}{(2\pi)^{1/2}} \int_0^y \exp(-x^2/2)\mathrm{d}x. \qquad (3)$$

Values of Eq. (2) are tabulated (e.g., Ref. 3). Each calculated value of d' is based on the results of at least 100 trials. The application of signal-detection theory to psychophysical testing is reviewed in detail by Green and Swets.[4]

Confidence intervals for each measurement of d' were calculated by using the delta method for asymptotic variance.[5] To test conformity to Weber's law, a chi-square test was applied to the results for each pattern to calculate the probability of the null hypothesis that the deviations from Weber's law are due to sampling errors. The square of the calculated confidence interval for each value of the ratio of unpaired points to paired points was used for the population variance. This is a more stringent test than the F test, because we are testing for similarity rather than for significant differences; the F test allows for variability in the estimation of population variance, resulting in higher p values. The constant in the formula derived from the model presented below was calculated for each pattern by linear regression through the origin.

RESULTS

Glass patterns of various types were chosen for study: translations, rotations, and dilations with varying spacings between point pairs. For a given type of pattern (e.g., translation through 9 min at 45 deg), the observer systematically studied the effect of varying the number of point pairs from 10 to 200 and varying the number of unpaired dots from 25 to 2000. Similar methods have been applied to the problem of

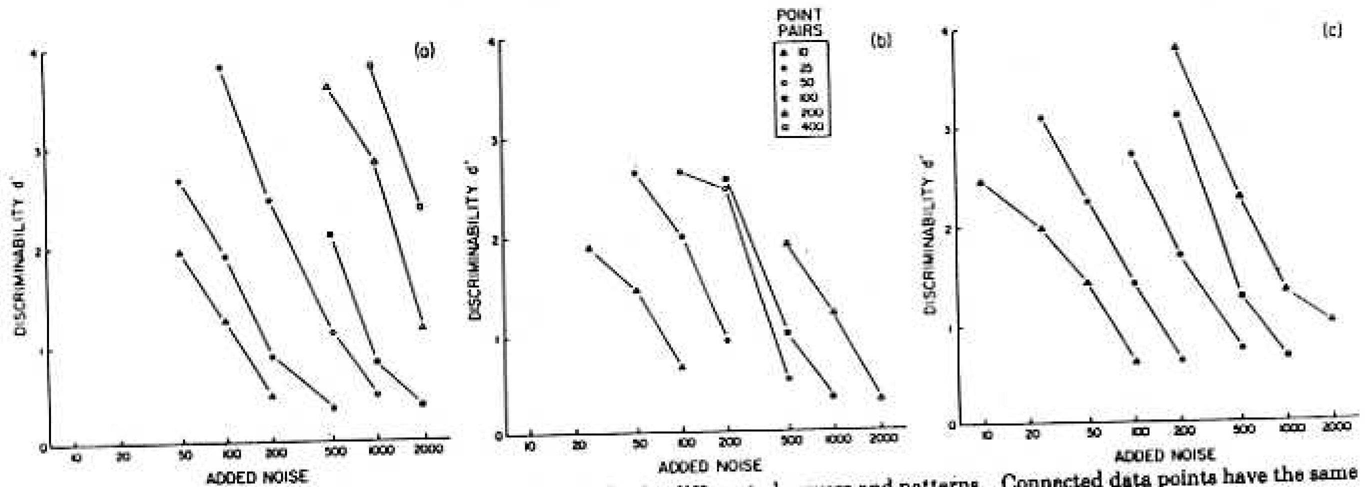

Fig. 2. Typical plots of discriminability d' as a function of noise for different observers and patterns. Connected data points have the same number of dot pairs. (a) Rotation of 9 min of visual arc (observer RKM). (b) Dilation of 18 min (observer HBB). (c) Translation of 18 min at 45 deg (observer GJM). As expected, d' decreases with increasing noise and increases as the number of point pairs increases.

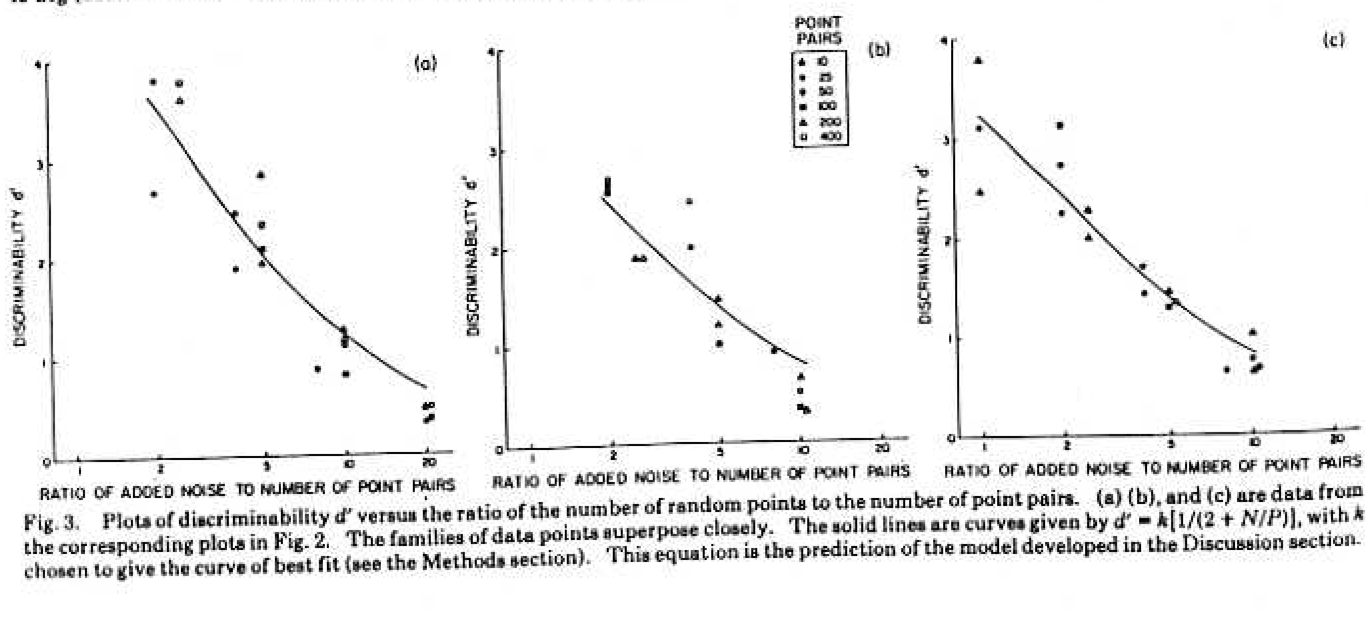

Fig. 3. Plots of discriminability d' versus the ratio of the number of random points to the number of point pairs. (a) (b), and (c) are data from the corresponding plots in Fig. 2. The families of data points superpose closely. The solid lines are curves given by $d' = k[1/(2 + N/P)]$, with k chosen to give the curve of best fit (see the Methods section). This equation is the prediction of the model developed in the Discussion section.

Table 1. Summary of Results[a]

Pattern Type	Observer	p	k	a (min^2)	Correlation Coefficient R
Rotation 9 min	RKM	0.54	14.3 ± 1.4	498	0.98
Rotation 9 min	GJM	0.02	12.9 ± 2.3	612	0.95
Translation 9 min at 45	HBB	0.30	12.0 ± 1.2	707	0.99
Translation 9 min at 0 deg	RKM	0.57	15.2 ± 1.5	440	0.99
Translation 9 min at 45 deg	RKM	0.04	18.3 ± 2.3	304	0.98
Translation 9 min at 90 deg	RKM	0.19	13.4 ± 2.9	567	0.96
Translation 9 min at 135 deg	RKM	0.35	14.3 ± 1.8	498	0.98
Translation 18 min at 45 deg	GJM	0.50	9.7 ± 0.8	1082	0.99
Dilation 18 min	HBB	0.92	9.8 ± 1.2	1060	0.98

[a] The p value represents the probability that deviations from Weber's-law behavior are the results of sampling errors. A p value of <0.05 indicates significant deviation from Weber's law behavior. k values are supplied with 95% confidence limits. For additional details, see the text.

symmetry detection,[6] quantum efficiency of light detection,[7] and sinusoidal signal discrimination in noise.[8]

Figure 2 is a typical graph of d' versus noise for a range of point pairs. As one would expect, d' decreases with increasing noise for a fixed number of point pairs and increases with increasing numbers of point pairs for a fixed level of noise. In Fig. 3, d' is shown as a function of the ratio of the number of unpaired dots to the number of point pairs for several observers and several types of patterns. Our results are summarized in Table 1. For most patterns, the family of curves relating d' to dot pairs and noise superimpose closely. This implies that $d' = f(N/P)$, where f is some function, N is the number of random dots, and P is the number of point pairs. Therefore the amount of noise that must be added to a pattern to maintain a constant level of d' is proportional to the number of dot pairs.

For two of the patterns studied, d' varied significantly ($p < 0.05$) from a function determined by N/P. For rotation through 9 min (observer GJM) the observer's performance on patterns with only 10 point pairs was relatively poor; performance with large numbers of point pairs was well determined by N/P. The same poor performance that occurred with 10 point pairs did not occur with other patterns. The results for translation through 9 min (observer RKM) were characterized by a relatively large spread of the data, but the model discussed below still described the results well, with a correlation coefficient of 0.98. Our statistical analysis does not enable us to say more about these deviations from the model.

DISCUSSION

For many tasks in many sensory modalities, the threshold for some percept varies in direct proportion to the intensity of the background on which the percept appears; this is Weber's law, an approximate empirical law described in all textbooks. Our finding appears at first to be just another example, but there is an important difference. For most of the tasks that have been investigated, the statistical limit to performance does not follow Weber's law but instead obeys the square-root law, as de Vries[9] and many others have pointed out. For our task, however, the statistical limit does follow Weber's law, essentially because we are looking for second-order structure in the image, not a first-order characteristic. Just as the rate of a second-order chemical reaction depends on the product of the concentrations of the two reactants, or on the square of the concentration if there is only one reactant, so the number of pairs of dots goes up with the square of the total number of dots; in our task we must pick out particular pairs against the background of fortuitous other pairs, and since the number of fortuitous pairs rises with the square of their density, the standard deviation of this number will rise in direct proportion.

If n is the total number of objects, then the number of background (irrelevant) pairings with the characteristic of interest is nearly proportional to n^2, so the standard deviation is nearly proportional to n. To remain distinguishable, the excess number of significant pairings present in the oriented pattern must increase in proportion to the standard deviation of the random pairings. Hence Weber's-law behavior is expected in this case. We now formulate these considerations in the form of an ideal model.

The method used by Barlow and Reeves[6] for calculating the ideal detection of mirror symmetry can be applied here. In order to determine whether an array of dots has the paired dots in random orientation or in the orientations expected from the type of displacement that was used to generate it, we must count the number of qualifying pairs. A qualifying pair is a pair of dots whose separation and orientation fall within some range of that expected from the displacement used, and we shall return to the question of what this range is. The numbers of such pairs at various orientations and separations represent the result of a two-dimensional autocorrelation on the pattern, and the allowable ranges described above determine the coarseness of the bins in this autocorrelation.

These ranges define an area a within which the second dot of a pair must fall for the pair to qualify and be counted. It is easy to see that the average number of qualifying pairs is given by the fraction of all pairs for which this condition holds, so this fraction is the ratio of the qualifying area a to the total area under consideration A. Call this ratio G; we can then show (see Appendix A) that the value of d' achieved by the model is

$$d' = 1/G^{1/2}(2 + N/P). \qquad (4)$$

In the simplest case we might consider the whole of the pattern at once, for this would be the most effective way of discriminating between oriented and unoriented arrangements of pairs. The solid curves in Fig. 3 are the predictions of this model, and Table 1 summarizes all our data. The model fits the data well but predicts slightly higher values of d' for $d' < 0.5$.

The constant k in Table 1 is the factor $1/G^{1/2}$ in Eq. (4) and hence can be used to estimate the actual value of the qualifying area, since we know the total area of the pattern. The results suggest values of 300–1000 min^2. On the other hand, some preliminary experiments were done in which the pairing of dots to produce Glass figures was done not precisely, but with a range of random variation, as in the experiments of Barlow and Reeves[6] on mirror symmetry. It was found that variations of ±2.5 arcmin in a separation of 12 arcmin and ±11 deg in orientation had only a small effect in diminishing the visibility of the patterns, and this suggests that the bins of an autocorrelation mechanism must have a qualifying area of only about 20 min^2. It should be noted, however, that any cause of efficiency loss would increase the estimate of bin size obtained by the calculations given in Table 1; this is because large bins would increase the numbers of fortuitous qualifying pairs and decrease the expected values of d', as any other cause of imperfect performance would. We must therefore ask in what ways the actual mechanism falls short of the simple model described above.

It is natural to suggest that the correlation is taken not over the whole pattern but over subregions; we are actually more or less forced to this conclusion because of the subjective appearance of the patterns, for it is evident that the orientation has one value in one region and another elsewhere. Indeed, we would not be able to distinguish the rotary and expansion patterns if this were not the case. We do not at this stage have enough evidence to elaborate a model along these lines, for we do not know how large the subareas are or how their outputs are combined to make an overall judgment as to whether they are oriented or random samples. Further experiments are required to enable us to decide this. Meanwhile, the constant k can be taken as an empirical measure of the intensity of the moiré effect for a given type of pattern; patterns more resistant to degradation by noise will yield higher values of k. Our results show that this constant is more or less independent of the number of point pairs in the pattern and the amount of noise added.

Stevens[2] noted that qualitative detectability continues even when each point has three or four unrelated neighbors closer than its paired point. In our experiment, detectability at a level of $d' = 1$ can occur when each paired point has 6–10 random neighbors closer than its mate. This argues against a detection mechanism in which local orientation at each point is determined by examining only its closest neighbor. The probability that points of a pair are nearest neighbors is given by a Poisson distribution, so if a nearest-neighbor mechanism is used, discriminability would drop exponentially with increases in the number of noise points rather than linearly as we found. A mechanism based on local autocorrelation seems necessary to explain this latter relationship.

SUMMARY

The following points have been made in this paper:

(1) The strength of the moiré effect elicited by a Glass pattern in the face of added noise follows Weber's law: the number of point pairs in the pattern must vary in proportion to the added noise to maintain a constant level of discriminability.

(2) Discriminability (d') is well predicted by a simple model based on local autocorrelation:

$$d' = k\,\frac{1}{2 + N/P}.$$

Our results are not consistent with a detection mechanism that examines each point's nearest neighbor.

(3) The constant k in the equation above can be used to estimate the size of the bins of the autocorrelation model, but it gives a value higher than that expected from other results if autocorrelation is performed over the whole pattern. We do not yet have results enabling us to specify the area over which autocorrelation is performed and the method of combination of results from subunits. Meanwhile, k can be regarded as a measure of the strength of the moiré effect of a given pattern that depends on the type of pattern, its size, the pair spacing, the observer, and possibly other variables. It is independent of the number of point pairs and the number of added random dots.

APPENDIX A

Suppose that there are N random dots and P pairs within an area A, the pairs in the stimulus pattern being oriented and separated according to some displacement rule and those in the random pattern being oriented at random but separated equally compared with those in the stimulus set. In order to determine whether an unknown pattern belongs to the stimulus set, each of the pairs must be examined to see if it qualifies as one of the deliberately generated oriented pairs, and the decision whether the stimulus is present should be based on the number of such qualifying pairs. Unlimited precision cannot be assumed, so we must accept a pair when the second dot falls within a certain qualifying area a whose position is determined by the first dot of the pair. For simplicity, put $G = a/A$, and note that this will usually be small. To obtain exact expressions for the means and variances of S and R, the numbers of qualifying pairs in the stimulus and random patterns, respectively, we should consider separately those pairs in which both dots come from the paired set and other pairs in which one or both dots come from the noise set. However, for present purposes we think that the following expressions are sufficiently accurate:

$$\text{mean}(R) = \text{var}(R) = G(N + 2P)^2,$$
$$\text{mean}(S) = P + \text{mean}(R),$$
$$\text{var}(S) = \text{var}(R) = G(N + 2P)^2,$$
$$\text{signal} = \text{mean}(S) - \text{mean}(R) = P,$$
$$\text{noise} = [\text{var}(R)]^{1/2} = G^{1/2}(N + 2P).$$

Hence $d' = P/[G^{1/2}(N + 2P)] = 1/[G^{1/2}(2 + N/P)]$.

These expressions are only approximate. The accurate ones are extremely complex and do not reduce simply to those used for calculating k in Table 1 and the curves in Fig. 3. For a future, more-accurate model we should note that var(S) is quite seriously wrong if we consider an area A that is small compared with the total area, for then the number of paired dots is subject to variability and will contribute to var(S). There are also other problems:

(1) The number of pairs is not $N^2/2$ but $N(N - 1)/2$; this could matter if a subarea contains only a few dots.

(2) Mean(S) is not right because the G factor applies not to all dot pairs but only to those that are not among the P. The true expression is $P(1 - G)$ + mean(R).

(3) Mean(R) is wrong because P pairs have a higher probability of qualifying than other pairs, since they must have only their orientation correct, not their orientation and separation.

(4) Severe problems occur at the edges of a pattern or of a subunit; for instance, with dilation there is a rim containing only dilated points.

The expressions given are reasonably good for A = total area and are thought to be adequate until a subregion model is developed.

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